CONVECTIVE HEAT TRANSFER IN SHORT CURVED CHANNELS

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Regions of different heat transfer regimes and the dependence of heat transfer coefficients on the length of a curved channel have been determined by an experimental investigation.

The main peculiarity of convective heat transfer in a curved channel is due to the field of mass centrifugal forces which causes the appearance of a vortex pair in the transverse section of the channel. For large heat fluxes and substantial channel cross-section dimensions an additional factor complicating the phenomenon will be thermal convection due to gravitational body forces. The complex nature of convective heat transfer, occurring here in conditions of coupled superposition of forced motion and secondary flows due to body forces, impels the investigator to turn to an experimental method.

The chief element of the experimental equipment (Fig. 1) is a horizontally positioned short curved channel ($l/d_e = 13.3$), with a smooth entrance, a square cross section of 49 × 49 mm, and a radius of curvature of the channel axis of R = 150 mm. The thin-walled plates forming the channel were of EI-437B steel and were jointed by means of epoxy adhesive. Water was supplied to the experimental section from a constant-level bath, its flow rate being determined by means of a measuring vessel downstream of the experimental section. Each plate forming the working channel had an electric heater with individually controllable current values. For equalization of temperature, a layer of copper 0.1 mm thick was electrodeposited on the plate surfaces on the heater side.

The heat transfer coefficient on the convex, concave, and flat surfaces forming the channel were determined by the gradient method.

On the surface of a curved or flat plate in which a two-dimensional temperature field exists, the mean heat transfer coefficient in a length $x \le l$, which corresponds to a central angle φ, is given by

$$\alpha = -\frac{\lambda}{\Delta t_m \varphi} \int_0^{\varphi} \left(\frac{\partial t}{\partial n}\right)_{n=0} d\varphi. \qquad (1)$$

In calculating the heat transfer coefficient α from (1) for a curved and a flat plate one needs to know the temperature fields $t = f(r, \varphi)$ and $t = F(y, \varphi)$. The equations of these fields were obtained by integration of the heat conduction differential equation, but their specific form is determined by the temperature distribution on the contours of the longitudinal section of the plates forming the channel. The mathematical basis for the gradient method for curved channels has been examined in [1, 2]*.

Temperatures were measured on two curved and one flat plate with the aid of nichrome-constantan thermocouples. The thermocouple junctions were in the form of disks of 0.2-mm diameter and were soldered into recesses in the wall so as to be flush with the surface. Thermocouple leads of 0.2-mm diameter were laid in grooves and anchored with epoxy adhesive. The number of thermocouples was 22 on the inside surface of each plate, 12 on the outer, and 5 each on the edges. The spacing of the thermocouples increased with increasing distance from the channel entrance. The measurement of temperature was made using an R2/1 potentiometer.

Two-dimensionality of the temperature field, i. e., absence of heat flux across the plate, was attained by means of thermal insulation of the side surfaces, and by compensating for the side heat loss by increasing the heater width in comparison with the plate width. At three sections of each plate six "tuning" thermocouples were attached over the width of the plate and were used to control the homogeneity of the temperature field in that direction.

The stream temperature was varied in the range 18°–35° C, that of the inside surfaces of the channel walls in the range 25°–60° C, and the stream velocity in the range 0.00167–0.351 m/sec during the experiments.

Reduction of the test results was done on a "Minsk" computer. The temperatures measured on the surfaces were approximated by power multinomials with the aid of orthogonal Chebyshev polynomials. A satisfactory approximation of the temperature relations was attained using multinomials of not more than the fourth order.

The integral appearing in (1) is expressed in terms of a sum of terms of an infinite series. A satisfactory accuracy in calculating this sum was reached with a number of terms not in excess of 60. Averaging of the heat transfer coefficient over the perimeter of the transverse section of the channel was accomplished according to the areas under the assumption that the two flat surfaces have the same heat transfer coefficient. The mean heat transfer coefficients were

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*In [2] the analytical solution of the temperature field problem was extended to the case of arbitrary heat transfer conditions on the faces of plates forming a channel.

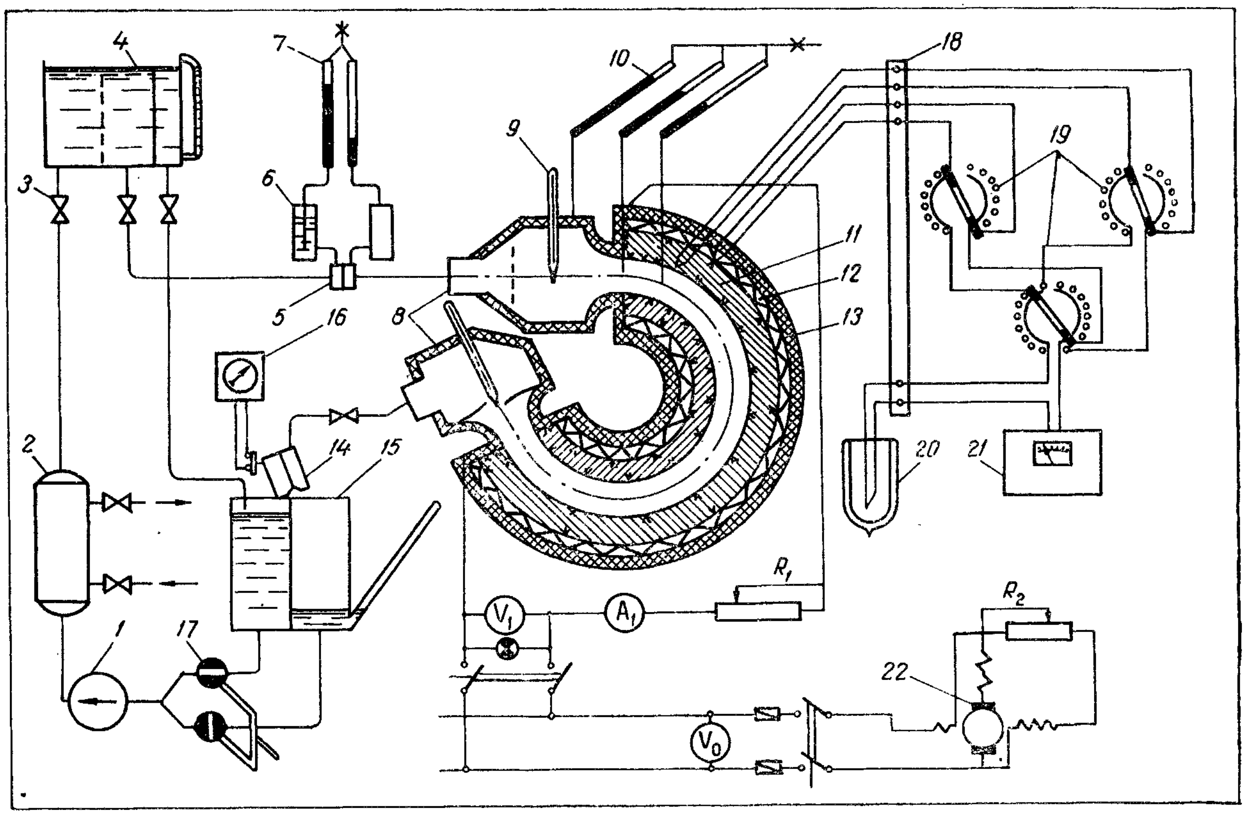

Fig. 1. Diagram of experimental apparatus. 1) Pump; 2) heat exchanger; 3) valves; 4) constant level tank; 5) measuring diaphragm; 6) shock absorbers; 7) piezometers; 8) damping and mixing chambers; 9) thermometers; 10) inclined piezometers; 11) experimental channel; 12) heater; 13) thermal insulation; 14) hopper; 15) measuring container; 16) electrical second meter; 17) ganged valve; 18) constant-temperature chamber; 19) switches; 20) Dewar flask; 21) potentiometer; 22) DC generator.

calculated for 12 values of x/d_e—0.52, 1.02, 1.6, 2.43, 3.21, 4.05, 4.8, 6.38, 8.05, 9.6, 11.2, 13.3.

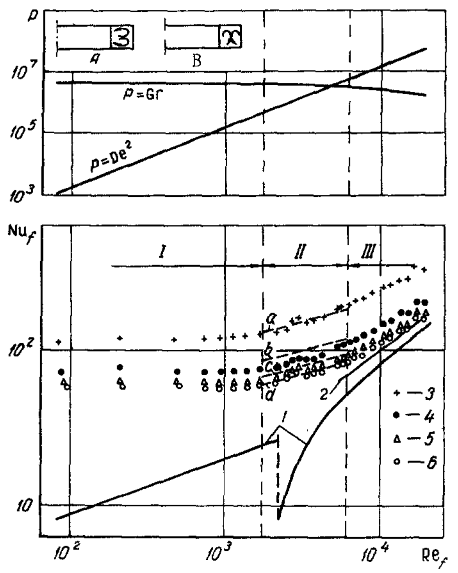

Fig. 2. The criteria Nu and P as a function of Reynolds number: 1) according to the formula for laminar, transitional, and turbulent flow in a tube; 2) according to Eq. (5); 3, 4, 5, 6) author's experimental data for x/d = = 1.6, 4.8, 9.6 and 13.3 respectively: dotted lines a, b, c, and d—according to the Meteis formula (14) for the same x/d values; I) region of thermal convection; II) region of mixed convection; III) region of laminar flow with macrovortices.

In processing the test data the mean fluid temperature was taken as a reference. The mixing temperature was measured at the entrance to and at the exit from the working section. In view of the small change in fluid temperature in the working section (2°–0.2°), a linear change of fluid temperature along the channel was adopted. The mean fluid temperature for a section of the channel was determined as the arithmetic mean of the temperatures at the ends of the section. The equivalent diameter was taken as a reference dimension.

Analysis of the differential equation of motion of a fluid in a body force field [3] has given a similarity criterion that reflects the effect of body forces on the stream:

$$P = \Delta F l^3/\rho\nu^2. \tag{2}$$

If a difference ΔF of mass forces arises in a gravitational force field from nonuniformity of density, then, taking the channel height $h = d_e$ as a reference dimension, we obtain

$$P = \frac{g d_e^3}{\nu^2}\beta\Delta t = \mathrm{Gr}. \tag{3}$$

In a centrifugal force field $\Delta F = \rho W^2_{max}/R$. For laminar flow $W_{max}/W = 2$. The distance between the points with maximum and minimum centrifugal force (h/2) is taken as a reference dimension. Allowing for this we obtain from (2)

$$P = \left(\frac{W d_e}{\nu}\right)^2 \frac{d_e}{D} = \mathrm{Re}^2 \frac{d_e}{D} = \mathrm{De}^2. \tag{4}$$

The gravitational and centrifugal forces lead to the appearance, in the transverse section of the curved channel, of paired vortices whose axes of symmetry are perpendicular.

The upper part of Fig. 2 shows the form of the secondary flows due to the centrifugal (A) and gravitational (B) forces. One of the force fields will therefore have a decisive effect on the heat transfer process, and the strength of their influence on the stream may be compared by comparing the Gr and De^2 numbers. The upper part of Fig. 2 shows the variation of Gr and De^2 with Re number observed in the experiments. The lower part of the figure shows the experimental results in the form of a correlation $Nu_f = f(Re_f)$ for four relative channel lengths. The Nu number was determined from the mean heat transfer coefficient for the channel. The relation $Nu_f = f(Re_f)$ is also given for a long straight tube, as calculated by the formulas of Mikheeva [4] (line 1), and for heat transfer in long coiled tubes under laminar conditions with macrovortices, obtained by reducing the test data from [5] and [6] (line 2)

$$\mathrm{Nu}_f = 0.0575\,\mathrm{Re}_f^{0.75}\,\mathrm{Pr}_f^{0.43}\,(d/D)^{0.21}\,(\mathrm{Pr}_f/\mathrm{Pr}_w)^{0.25}. \tag{5}$$

This formula was obtained by putting the test data into the form $K_f = f(\mathrm{De})$ with $\mathrm{De} = 26\text{–}7 \cdot 10^3$, where

$$K_f = \frac{\mathrm{Nu}_f}{\mathrm{Re}_f^{0.33}\,\mathrm{Pr}_f^{0.43}\,(\mathrm{Pr}_f/\mathrm{Pr}_w)^{0.25}} = \frac{\mathrm{Nu}_f}{\mathrm{Nu}_{f_0}}. \tag{6}$$

An investigation was made in [5] of heat transfer in long coiled tubes using water as the heat transfer agent and $D/d = 6.2\text{–}23.8$, $Re_f = 2 \cdot 10^3\text{–}2.5 \cdot 10^4$, $l/d > 60$. In [6] the coiled tubes were investigated with $D/d = 23\text{–}62.5$, $Re_f = 63\text{–}2.1 \cdot 10^4$, $l/d > 218$. The heat transfer agent was three kinds of liquid with Pr = = 7–369.

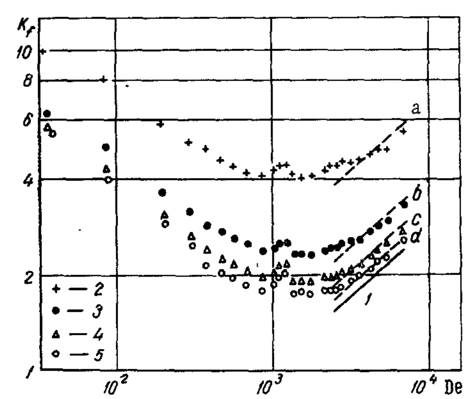

Fig. 3. Dependence of K_f on De number: 1) from Eq. (5); 2, 3, 4, 5) author's experimental data with x/d = 1.6, 4.8, 9.6, and 13.3 respectively: dotted lines a, b, c, and d are from Eq. (16) for the same x/d values.

Formula (5) was obtained at small Gr and may be used whenever the effect of that parameter on the heat transfer may be neglected.

Figure 3 shows the relation $K_f = f(\mathrm{De})$ for various relative channel lengths, and gives also for comparison the line 1, constructed from (5), which it is convenient in this case to write in the form

$$\mathrm{Nu}_f = 0.0575\,\mathrm{Re}_f^{0.33}\,\mathrm{De}_f^{0.42}\,\mathrm{Pr}_f^{0.43}(\mathrm{Pr}_f/\mathrm{Pr}_W)^{0.25}. \quad (7)$$

Analysis of Figures 2 and 3 enables us to determine the heat transfer regimes.

It may be seen from Fig. 2 that for $\mathrm{Re}_f \le 1700$ the heat transfer intensity is independent of Re, and therefore Nu in this region is determined exclusively by the thermal convection while $\mathrm{De}^2 \ll \mathrm{Gr}$. The thermal convection region is designated by the numeral I in the figures.

Figure 3 shows that for Re = 1700–6000 (De = = 690–2400) the De number, and therefore also the centrifugal forces, have practically no influence on the heat transfer intensity. Figure 2 shows that in the major part of this region Gr is considerably larger than De^2, the two quantities only becoming comparable in the right side of the region. For this reason it may be asserted that region II is a mixed convection region in which the heat transfer is determined by the simultaneous influence of forced and free convection on the stream.

With further increase of Re the heat transfer intensity increases with increase of De (Fig. 3), as $\mathrm{De}^2 \gg \mathrm{Gr}$, and therefore the effect of thermal convection on the heat transfer may be neglected. As is known from analysis of flow structure in coiled tubes, there will be laminar flow of the fluid with macrovortices (Region III) in this region of variation of Re and De (Re > 6000, De > 2400). The upper boundary of this region may be estimated by Aronov's formula, obtained for long coiled tubes of circular section [7]:

$$\mathrm{Re}_{\mathrm{cr}} = 18\,500\,(d/D)^{0.3}. \quad (8)$$

For the curved channel investigated ($D/d_e = 6.1$), a value $\mathrm{Re}_{\mathrm{cr}} \cong 1.1 \cdot 10^4$ was obtained from (8). This number may change substantially for short channels, however. In the present tests (Re_f up to $2 \cdot 10^4$), we were not able to determine the upper boundary of region III sharply.

The boundary established in the present investigation for the thermal and mixed convection region in a horizontal tube is not universal. For $(\mathrm{GrPr})_f = 3.1 \cdot 10^7$ ($\mathrm{Gr}_f = 4.5 \cdot 10^6$) the mixed convection region occurs for $\mathrm{Re}_f \ge 1700$. For other values of $(\mathrm{GrPr})_f$ special experiments are required to discover the boundaries of this region.

The boundary of the regions of mixed convection and of laminar flow with macrovortices due to centrifugal forces may be found approximately from the condition $\mathrm{Gr}_f = \mathrm{De}_f^2$.

The literature contains no information on convective heat transfer in a fluid flowing through a horizontal tube, under conditions when the heat transfer is governed by thermal convection.

Therefore, in order to generalize the experimental data on the effect of channel length on heat transfer in the thermal convection region I, the results were compared with Oliver's formulas [8] for mixed convection in laminar flow, and with those of Mikheeva [4] for free convection around the outer surface of a horizontal tube. For mixed convection in a horizontal tube with $l/d \ge 70$, Oliver obtained the formula

$$\mathrm{Nu}_f = 1.75\,[\mathrm{Gz}_f + 0.0083\,(\mathrm{Gr\,Pr})_f^{0.75}]^{1/3}\,(\mu_f/\mu_W)^{0.14}. \quad (9)$$

When the influence of forced motion on the heat transfer is removed, $\mathrm{Gz} \to 0$, and (9) takes the form

$$\mathrm{Nu}_f = 0.35\,(\mathrm{Gr\,Pr})_f^{0.25}\,(\mu_f/\mu_W)^{0.14}. \quad (10)$$

Mikheeva's formula for free convection around the outer surface of horizontal tubes has the form

$$\mathrm{Nu}_f = 0.51\,(\mathrm{Gr\,Pr})_f^{0.25}\,(\mathrm{Pr}_f/\mathrm{Pr}_W)^{0.25}. \quad (11)$$

The upper part of Fig. 4a shows experimental points for three values of Re typical of the I region. Line 1 corresponds to (10), and line 2 corresponds to (11). It may be seen that the experimental heat transfer results are in good agreement with Mikheeva's formula. The lower part of Fig. 4a shows the relation $\mathrm{Nu}_f/\mathrm{Nu}_{f_0} = f(x/d_e)$, the quantity Nu_{f_0} used in the construction of which being calculated according to (11). Approximating the test data we obtain the formula

$$\mathrm{Nu}_f/\mathrm{Nu}_{f_0} = 0.91 + 2.95\,(d_e/x)^{0.83}, \quad (12)$$

corresponding to line 6 on the figure.

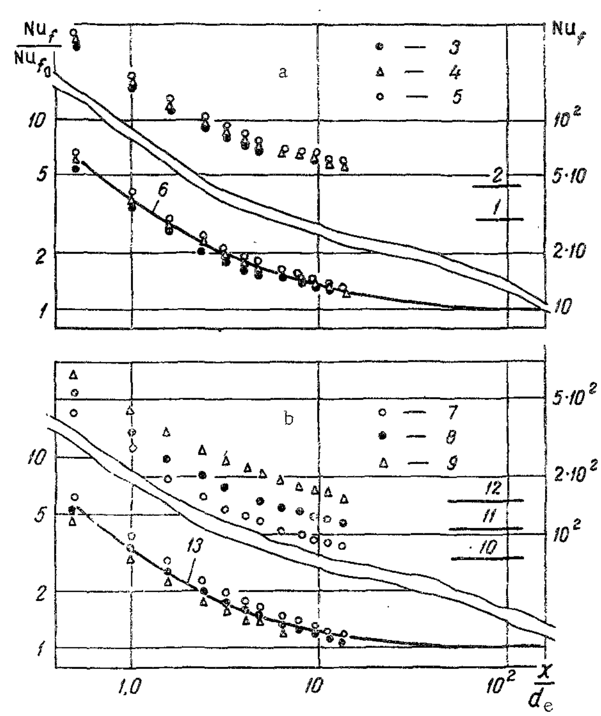

Fig. 4. Comparison of the experimental and calculated data in Regions I (a) and III (b) of the flow: 1) from Eq. (10); 2) from Eq. (11); 3, 4, 5) experimental data of the author for Re_f = 88, 740, and 1705 respectively; 6) from Eq. (12); 7, 8, 9) author's experimental data for De_f = 2556, 4260, and 6969; 10, 11, 12) from Eq. (7) for the same values of De; 13) from Eq. (15).

Thus, for short horizontal tubes and channels (straight and curved), the coefficient of heat transfer between the fluid and the wall, under conditions when thermal convection plays an important part, may be calculated from the formula

$$\mathrm{Nu}_f = 0.51\,(\mathrm{Gr\,Pr})_f^{0.25}\,(\mathrm{Pr}_f/\mathrm{Pr}_W)^{0.25}\,[0.91 + 2.95\,(d_e/x)^{0.83}]. \quad (13)$$

The experimental data on heat transfer in mixed convection (Region II) have been compared with Meteis' formula which was obtained for straight horizontal tubes in turbulent flow:

$$\mathrm{Nu}_f = 4.69\,\mathrm{Re}_f^{0.27}\,\mathrm{Pr}_f^{0.21}\,\mathrm{Gr}_f^{0.07}\,(d/x)^{0.36}. \quad (14)$$

The dotted lines a, b, c, and d in Fig. 2 correspond to Eq. (14) for the values of Gr and Pr occurring in the tests and with $x/d_e = 1.6,\ 4.8,\ 9.6,\ 13.3$.

It may be seen that Eq. (14) is roughly a correct description of the effect of relative channel length on the intensity of heat transfer in the conditions under examination.

The reduction of the test data for the region of laminar flow with macrovortices (III) was performed under the assumption that the heat transfer coefficient in a long channel of square section may be described by Eq. (7).

The upper part of Fig. 4b shows the experimental points defining the relation $\mathrm{Nu}_f = f(x/d_e)$ for three values of De, while the lines 10, 11, and 12 have been calculated for these same De values according to (7). Assuming that (7) is valid for $x/d_e \geq 50$, the test data examined above may be used to obtain a correction to the length required for calculating heat transfer in short curved channels.

The lower part of Fig. 4b shows the relation $\mathrm{Nu}_f/\mathrm{Nu}_{f0}$, for which the quantity Nu_{f0} has been determined from (7). This relationship is described well by the formula

$$\mathrm{Nu}_f/\mathrm{Nu}_{f_0} = 0.96 + 2.41\,(d_e/x)^{1.025}, \quad (15)$$

corresponding to line 13 in Fig. 4b.

Therefore, for laminar flow with macrovortices, the heat transfer in short curved channels may be calculated from the formula

$$\mathrm{Nu}_f = 0.0575\,\mathrm{Re}_f^{0.33}\,\mathrm{De}_f^{0.42}\,\mathrm{Pr}_f^{0.43} \times \left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_W}\right)^{0.25}\left[0.96 + 2.41\left(\frac{d_e}{x}\right)^{1.025}\right]. \quad (16)$$

The dotted lines a, b, c, and d in Fig. 3 correspond to (16) for four values of x/d. It may be seen that the agreement with the experimental data is fully satisfactory right up to $D_{ef} = 7 \cdot 10^3$, which corresponds in the experiments to a value $R_{ef} = 1.7 \cdot 10^4$.

NOTATION

D is the diameter of curvature of the channel axis; d is the tube diameter; d_e is the equivalent diameter; ΔF is the difference in the mass forces; g is the acceleration due to gravity; h is the channel height; K_f is a coefficient defined by Eq. (6); l is the characteristic dimension, the channel length; n is the normal to the heat transfer surface; P is a parameter describing the influence of mass forces on the stream of fluid; R is the radius of curvature of the channel axis; t is the temperature; Δt_m is the mean temperature head; $(\partial t/\partial n)_{n=0}$ is the normal temperature gradient at the heat transfer surface; W is the flow velocity of the fluid; W_{max} is the maximum velocity in the channel section; x, y, and r are coordinates; Gr is the Grashof number; Gz is the Graetz number; Nu is the Nusselt number; Pr is the Prandtl number; Re is the Reynolds number; De is the Dean number; α is the mean heat transfer coefficient; β is the coefficient of volume expansion; λ is the thermal conductivity of the wall material; μ is the dynamical viscosity; ν is the kinematic viscosity; ρ is the density; φ is the angle at the center corresponding to a channel length x. The subscripts f and W describe physical parameters evaluated at the temperature of the fluid and of the wall, respectively.

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